



Advisory
Committee on
Mathematics
Education

ACME Position Statement on Qualifications in Mathematics at Level 3 from 2011 February 2009

A good mathematical education is vital both for society as a whole – for its industry, its commerce, its decision making and its cultural and intellectual richness – and also for individual citizens for whom it can open doors to a more rewarding and fulfilling life. Mathematics has a key role not only in the physical sciences, technology and engineering but also in a wide range of other subjects, including economics, business and finance, ICT, social science, health and bio-medical sciences, architecture, design, geography and environmental studies. Additionally studying mathematics develops more general intellectual and thinking skills which transfer to other activities in a valuable way. Virtually no one is untouched by mathematics, yet many learners fail to see its value, or underestimate its rewards, and thus avoid its study. As a result, much potential is left undeveloped and the intellectual capital on which this country depends for its future prosperity is diminished. This situation and the issues around it were analysed in *Making Mathematics Count*¹ which called for the development of new pathway models and approaches for 14–19 mathematics education (Recommendation 4.11, page 104).

ACME² is currently engaged in a project on Level 3 Mathematics which seeks to enhance the committee's contribution to discussions on the mathematical pathways to be adopted for first teaching from 2011 and to offer a vision for Level 3 Mathematics education in the latter part of the next decade. The project has been taken forward by an ACME working group.

With decisions about the 2011 curriculum being likely to be made in 2009, that part of the work has been addressed first, and is the subject of this statement, which aims to contribute to the process leading to those decisions. The proposals are made in the context of the work being undertaken by QCA and others on developing mathematical pathways; so that they focus primarily on qualifications and assessment.

In addition to discussions inside and outside ACME, the position set out here is informed by a consultative workshop (during which we heard conflicting views), published research and some small-scale exploratory research commissioned by ACME.

Currently most study of mathematics at Level 3 leads to AS or A GCE. Given the remarkable improvement in uptake since the introduction of the current specifications, it is understandable that many will feel that no further change should be introduced in 2011. However the enquiries and deliberations of the working group on this matter led to the conclusion that leaving the qualifications unchanged for a dozen years in an ever-changing world would not be the best way forward.

In the statement our recommendations are listed first, followed by the rationale for them. The section of the rationale relating to each recommendation is indicated in parenthesis at the end of the recommendation. A short historical and statistical appendix is provided at the end of this document.

Recommendations

- 1 ACME recommends that all young people should have the opportunity, and should be encouraged, to continue to progress in their learning mathematics throughout their 5–19 education. (1.1)

There is a need for Level 3 qualifications smaller in volume than an AS GCE to encourage fewer learners than at present to drop mathematics at age 16 only to find themselves needing it later in employment or higher education. (2.2)

Free Standing Mathematics Qualifications should continue to form part of the qualification offering at Level 3. (2.3)

¹ *Making Mathematics Count*: The Report of Professor Adrian Smith's Inquiry into Post-14 Mathematics Education, February 2004.

² ACME is an independent committee which was established by the Royal Society and the Joint Mathematical Council, with the support of all major mathematics organisations, to act as a single voice for the mathematical community. It advises government on issues relating to 5–19 mathematics education in England.

Students should be required as a minimum to pass a qualification in functional mathematics or an equivalent at level 3 in order to gain a level 3 diploma in any field, and hence to develop, broaden and deepen the mathematical skills they have at age 16. (2.5)

- 2 ACME recommends that, in order to align with changes to Key Stage 4 of the National Curriculum and GCSE from 2010, changes to existing qualifications should be deferred from 2011 to 2012. (1.4)
- 3 ACME recommends that AS GCEs and A GCEs should be available in Mathematics, Further Mathematics and Use of Mathematics. Pure Mathematics should be withdrawn as a separate A GCE. (3.2, 3.11 and 3.14)
- 4 ACME recommends that GCE Mathematics and Further Mathematics should place greater emphasis on problem-solving, modelling, and construction and communication of rigorous mathematical arguments. The regulatory authorities must take steps to ensure that this happens. (3.8)
- 5 ACME recommends that consideration should be given to redesigning A GCE Mathematics so that it consist of four units, each assessed by a timed written paper:
 - An AS pure mathematics unit (with a weighting of 30%),
 - An AS applied mathematics unit composed equally of mechanics and statistics (with a weighting of 20%),
 - An A2 pure mathematics unit (with a weighting of 30%),
 - An A2 applied mathematics unit composed equally of mechanics and statistics (with a weighting of 20%),
 - The content of each unit should be specified in the subject criteria. (3.5 to 3.9)

The requirement that one unit should be assessed without access to a calculator should be removed. (3.9)

- 6 ACME recommends that A GCE Further Mathematics should continue to consist of six units. It should retain something approaching the present flexibility and amount of choice of units but consideration should be given to developing pure mathematics cores at AS and A2. (3.12)

The Further Mathematics Network should retain its role in ensuring all learners have access to GCE Further Mathematics. (3.13)

Note: Recommendations 5 and 6 were not agreed unanimously by the committee. There was one strongly dissenting voice; this member's views are expressed separately in an annexe to this position statement. The other members were strongly in favour of these recommendations; and whilst conceding there were risks involved they thought that the benefits of adopting the recommendations outweighed the risks of not adopting them.

- 7 ACME recommends that A GCE Use of Mathematics is introduced in a manner conducive to ensuring a gradual uptake of the subject. (3.17)

GCE Use of Mathematics should be presented as a separate qualification from GCE Mathematics with its own assessment objectives and its development tailored to a different set of users. It has to be distinctive so as not to remove learners from the pool of those with A GCE Mathematics when that qualification is required for the study of mathematics, the physical sciences and engineering in higher education. (3.17)

It is vital that appropriate Information, Advice and Guidance are available to learners when choosing between mathematical qualifications. (3.17)

Further consideration should be given to determining the most appropriate combination of examinations and portfolios in the assessment of the units in GCE Use of Mathematics. (3.17)

All these previous recommendations will depend on a review of A GCE Use of Mathematics in 2013 to (a) make sure that it is not undermining the numbers doing A Level Mathematics and Further Mathematics and (b) ensure that lessons learned from A GCE Use of Mathematics are used to improve A Level Mathematics and Further Mathematics.

- 8 A GCE qualification should be developed in statistics which is independent of GCE Mathematics and GCE Further Mathematics. (3.2)
- 9 ACME recommends that there should be a qualification additional to GCE which provides stretch, challenge and inspiration for the most mathematically able learners, and acts as an effective measure of

aptitude to pursue mathematically highly demanding degree courses. A GCE Mathematics cannot on its own do this. The new qualification should be aimed at all A grade GCE Mathematics candidates. It should have a grading scale which recognises achievement at a broader range of levels than the AEA. The qualification must be awarded UCAS points to ensure take up and currency. (4.4)

Until the new qualification is in place, AEA Mathematics should remain available. (4.4)

- 10 ACME recommends that, whilst acceleration may be appropriate for a few learners, most of the most able learners in mathematics should deepen and extend their mathematics at Level 3 rather than proceed prematurely to Level 4. Centres should include enrichment activities within their provision as well as preparation for qualifications; these activities can be motivating and enriching for many learners, not only the most able. (4.5)
- 11 ACME recommends that professional development, co-ordinated by the National Centre for Excellence in the Teaching of Mathematics, should be provided to support teachers in delivering the changes it has recommended. (3.10)
- 12 ACME recommends that exploratory work should begin in preparation for the next generation of qualifications. It should include a possible realignment of GCE qualifications, combining Mathematics and Use of Mathematics, and the possible use of a wider range of assessment styles than timed written papers. (3.18)

Rationale

1 Shaping a vision for the future

- 1.1 ACME believes that all young people should learn mathematics throughout their 5–19 education; such a situation is common across the developed world and is one to which this country should aspire. In an increasingly quantitatively-oriented world, we shall find ourselves left behind, as individuals and as a country, if we do not develop our mathematical abilities to the full. Although we now have universal participation in the learning of mathematics 5–16, that is not the case 16–19 and what is more we are not in a position to make it such in the short to medium term. What ACME believes can and should be done, at this time, is to ensure that all 16–19 learners have the opportunity to continue to develop their mathematical abilities from wherever they are at age 16. For learners who have not yet achieved a qualification at Level 2 of the National Qualifications Framework, such provision is already government policy; this paper is concerned with developing provision at Level 3.
- 1.2 In the summer of 2007 ACME decided to make the development of its thinking on provision for mathematics at Level 3 a priority for the proactive side of its work. In January 2008 ACME published a position statement on Mathematics and Level 3 Diplomas; this paper provides a road map for general qualifications in Mathematics at Level 3 from 2011 until the implementation of any changes ensuing from the review of qualifications presently scheduled for 2013, and it is our intention to publish in the spring of 2009 a paper outlining a vision for mathematics education subsequent to that review. To help inform its thinking leading to this position paper, ACME held a consultation workshop with the mathematics community in June 2008.
- 1.3 In undertaking its work, ACME has striven to make proposals which would:
- better prepare all learners for the needs of further education, higher education, the workplace and the needs of life (1.1, 2.1, 3.1 and 4.1);
 - provide a suite of qualifications that is valued and understood by learners, their carers, those working in education (including higher education), employers and other users (2.2, 2.3, 2.4, 3.2, 4.4);
 - raise the proportion of 16-19 year olds continuing to study mathematics, and proceeding beyond Level 2 (2.1 to 2.4, 3.2, 3.16, 3.17);
 - raise the proportion of 16-19 year olds studying for mathematics degrees and ensure they are better prepared for such study (3.5, 3.8, 4.3, 4.4);
 - raise the proportion of 16-19 year olds studying for other numerate degrees and improve their preparation for such study (3.2, 3.5, 3.8, 3.16, 3.17);
 - ensure that all learners, including the most able, can fulfil their potential in mathematics (4.3 to 4.5);
 - provide a long term, overarching view with broad perspective, informed by experience of delivering and engagement with all aspects of Level 3 mathematics (1.1 and 1.2);
 - mindful of possible future curricular and assessment structures, provide an overview of a set of mathematical pathways which meet the needs of learners, at Level 3, and are viable to deliver, building on successful practice in this country and elsewhere (1.1 and 1.2);
 - help shape the present Pathways developments to act as steps towards achieving the above benefits (1.4 to 1.6).
- 1.4 As remarked in the introduction, key decisions shaping provision from 2011 will be taken in 2009; this paper is intended as a contribution to the process leading to those decisions. The proposals are made in the context of the work being undertaken by QCA and others on developing mathematical pathways; so they focus primarily on qualifications and assessment. Also, the proposals assume as the starting point for provision at Level 3 the current situation at Level 2 and the development over the next few years of a pair of GCSEs in mathematics. This is regarded as an essential foundation for all that follows regarding level 3 in the discipline. So that it is possible to build on the changes in Key Stage 4 of the National Curriculum and GCSE which are being introduced from 2010, yet not disadvantage those in the previous cohort, the changes to GCE Mathematics and Further Mathematics should be deferred from 2011 to 2012; the development of new qualifications to support Diplomas should nevertheless continue on the existing timescale. Further, the underdeveloped nature of the evidence base available is seen as rendering imprudent more radical changes from 2011 or

2012. (It is not too soon to begin exploratory work on ideas for provision from 2016³, if sufficient time is to be available to provide an adequate evidence base for decision making about that provision.)

1.5 Level 3 qualifications are predominantly used by 16–19 year old learners, and the needs of those young people have been at the forefront of ACME’s considerations, although it realises that such qualifications will also be used by adult learners and there are circumstances in which it can be appropriate for qualifications at this level to be used with younger learners. ACME’s brief extends only to England but it recognises that provision in England does not take place in isolation, as many qualifications are shared with Northern Ireland and Wales, and Level 3 qualifications are stepping stones to higher education throughout the United Kingdom.

1.6 The qualifications structure adopted should further the aims of the curriculum rather than determine the curriculum. It should also be of value to and have the confidence of all user groups: learners, their carers, their teachers, employers, further and higher education, funders and the general public.

Attention is drawn to the ‘Principles for assessment in mathematics’ and Recommendations in the ACME Report *Assessment in 14–19 Mathematics* published in 2005 and to ACME’s submission to the House of Commons Select Committee on Education and Skills’ Inquiry into Assessment and Testing in 2007.

1.7 These proposals have benefited from knowledge of the piloting currently being undertaken, as well as existing and previous specifications. It is important for the success of future developments, especially those following the 2013 review of qualifications, that curriculum development work continues to be undertaken. In the not so distant past several valuable innovations were the result of initiatives developed by practitioners and curriculum development projects such as MEI and SMP. ACME would wish to see such initiatives more actively encouraged and supported in the future⁴ so that mathematics education can continued to be refreshed and attuned to the needs of the society it serves.

2 Modern Society is Mathematical: we all need mathematics

2.1 Increasingly, mathematical ideas and quantitative data underlie decision-making, not just in the natural sciences, technology and engineering but also in business, in the social and medical sciences, in the community and in personal activities. In most developed countries it is taken for granted that mathematical education will continue until the end of schooling. In England, we are in danger of finding our young people at a disadvantage in coping with the modern world and competitor economies having a competitive edge over our own.

2.2 About 10 to 15% of 16–19 year old learners whose programmes are principally at Level 3 already study mathematics, from interest or from recognising its utility for work or study, now or in the future. Some will still be working towards a qualification at Level 2. Some, whilst having achieved a Level 2 qualification in the subject, will benefit from continuing to develop their mathematics at that level. But there remain many learners who achieve well in mathematics at Level 2 at age 16 and then give up study of the subject; typically these learners may be taking programmes of GCEs in non-mathematical subjects or an Advanced Diploma which does not contain any mathematical units (beyond the requirement to achieve Level 2 Functional Mathematics⁵). Many of these learners will proceed to higher education where a significant proportion will take courses which involve mathematics to some degree, something many find challenging following two years during which their mathematical competence has atrophied. There is a need for one or more qualifications which will sustain and develop their mathematical learning yet are small enough to fit within their programmes of study.

2.3 There exist several Level 3 Free Standing Mathematics Qualifications (FSMQ) which typically need 60 guided learning hours (glh) of study. They have been used to provide and accredit mathematical learning in support of another subject, for example, the FSMQ in Using and Applying Statistics can be a valuable support for those studying GCE Geography or Psychology. FSMQs also act as units within GCE Use of Mathematics. Assessment is by a written examination and a portfolio; the impending removal of the portfolio is likely to lead to a narrowing of the learning experience and a diminution of the educational value of the qualification. (The Level 3 FSMQ in Additional Mathematics which has

³ For ease of reference, the changes ensuing from the review of qualifications presently scheduled for 2013 will be referred to in this paper as changes from 2016, that being the earliest year in which it would be practical to implement any such revised provision.

⁴ At present, the possibility of developing a pilot qualification is not widely publicised although it is to be found in *Pilot qualifications: Guidance for awarding bodies* on which is available on the Ofqual website.

⁵ Within a few years, most learners will have obtained Level 2 Functional Mathematics, either as part of a Level 2 Diploma or GCSE, before they start their Level 3 courses; those who have not are likely to obtain it soon after the start of the course.

only a written examination has a distinctive role; it is primarily used by more able learners in Year 11 in some schools as a preparation for and foretaste of GCE mathematics.) Although the uptake of these qualifications is not large⁶, users of these qualifications value them highly and are enthusiastic in their support for them. We see FSMQs continuing to be a useful part of the qualifications offering at Level 3 and there may be advantage in the development of further such qualifications, especially as part of the Additional and Specialised Learning (ASL) in some Advanced Diplomas, but they do not provide the complete answer.

- 2.4 It is essential that all learners on Level 3 courses have the opportunity to progress with mathematics. There therefore needs to be a minimum requirement to pass a Level 3 Functional Mathematics qualification or an equivalent available for those who do not choose to carry on to GCE mathematics or to specialist diploma units in mathematics or FSMQs as part of their Level 3 course. Its main function would be to support the progress in mathematics of students who are not likely to be specialising in numerate subjects but may well need a basis of mathematics for higher education in subjects like nursing or primary education, or in employment routes, for example, construction.
- 2.5 There has been debate within the mathematical community about whether there should be a single qualification in Functional Mathematics at Level 3 or whether there should be a variety of qualifications fulfilling this role, each tailored to a different line of learning. Whilst we can see attractions in developing such tailored qualifications, especially in the longer term, ACME believes it to be an urgent priority that a single generic Functional Mathematics qualification be developed at Level 3, so all learners can have access to such provision. With Advanced Diploma courses starting this September, there is pressing need for this qualification and we would hope that it would be available, at least as a pilot, by 2010.
- 2.6 It needs to be recognised that learners will start and may well finish their studies at different stages of mathematical development but the qualification should be such as to recognise two years' progress by a learner working at this level. The pass standard should be set at a level to which it was realistic that most learners could aspire but the material covered should provide opportunities for challenge. We would expect the course to take up something like 60 glh spread over two years (that is roughly 1 hour per week).
- 2.7 The wide range of possible future mathematical needs of students, and the wide range of starting points (anything from adult diploma students with a scraped Level 2 in Functional Mathematics and no GCSE to students with A* at GCSE who have elected not to take GCE mathematics) provide major challenges in the design of the qualification. The single syllabus would need to include material which is useful to a wide range of students although any particular student might find parts of it which are not immediately relevant, there should be much opportunity to relate the material to learners' other studies through appropriate contextualisation. The content would draw on and develop content from Level 2 mathematics but with an applied slant and much greater use of ICT. This should make it challenging for students at different entry levels. (It might be necessary to introduce a few new topics for coherence but they should be very limited.)
- 2.8 A course for Level 3 Functional Mathematics might include:
 - developing fluency with a spreadsheet programme, including formulae, sorting functions, statistical displays and data functions like median, mean, range and quartiles;
 - interpretations of quite complex statistical data presented in diagrams and tables;
 - modelling using simple functions, including the use of computer algebra and graphing software;
 - financial applications, including rates of change (but not calculus);
 - probability and risk in realistic situations;
 - perhaps some three-dimensional work as used in design settings, for example, use and interpretation of a computer-aided design package.

It is very difficult to see how this could be assessed without the use of computers, either in timed assessments or in work contributing to a portfolio, with questions in the form of long tasks.

The development of such a qualification does not mean working on a totally green-field site; lessons can be learned from the positive and negative experience with existing qualifications, such as Key Skills.

⁶ FSMQs were first awarded in 2001. At Level 3, except for Additional Mathematics which is awarded by OCR, all awards are now made by AQA. In 2007, AQA received 8527 entries at all levels, including 3490 entries at Level 3 (AQA website, Provisional Results for June 2007). It should be noted that some learners, especially those taking ASGCE Use of Mathematics, will have made more than one entry.

- 2.9 The Level 3 qualification in Functional Mathematics should, along with other mathematical qualifications at this level, enable there to be an entitlement for 16–19 learners who have achieved a Level 2 qualification in mathematics to study the subject at Level 3, with the long-term goal of participation by all such learners. Thought should also be given to how a Level 3 Functional Mathematics qualification might, in the future, act as step towards more mathematical study and contribute towards a larger mathematical qualification, such as an ASGCE, at this level.

3 Keeping it simple

- 3.1 Whilst we wish to see as many learners as possible incorporating some mathematics into their studies, there will be many learners for whom it will be appropriate and desirable that they incorporate more extensive mathematical courses into their programmes. Further, we wish to see the numbers taking such larger courses increase substantially. Some will undertake that study through units of Principal Learning or tailored units of ASL within Advanced Diplomas whilst others will take ASGCEs and AGCEs (which may also be part of a Diploma's ASL); it is with GCE provision that we deal here.

- 3.2 Several mathematical ASGCE and AGCE subjects exist at present:

- Mathematics
- Further Mathematics
- Further Mathematics (Additional Award)
- Pure Mathematics
- Statistics
- Use of Mathematics⁷

Whilst this is a reduction from the number of titles initially offered under Curriculum 2000, we believe that the mathematical offering can, with advantage, be simplified further to consist of ASGCEs and AGCEs in:

- Mathematics
- Further Mathematics
- Use of Mathematics
- Use of Statistics

The provision for Further Mathematics should provide recognition for those learners offering nine or twelve units, as well as those offering three or six units.

The provision for Statistics should be independent of the provision for Mathematics and Further Mathematics and not share units with it, although there will inevitably be some overlap of content. (Although at first sight sharing modules between Statistics and Mathematics seems a useful economy, in fact the more mathematical and theoretical approach from which those taking Mathematics can benefit is not easily combined with the more practical approach which is appropriate for Statistics.) In view of its more practical approach it may be possible for Statistics, as well as Use of Mathematics, to make use of FSMQs. To emphasise the applied nature of the qualification, there may well be merit in calling it Use of Statistics. These qualifications would be of particular use to learners studying for Advanced Diplomas who might not otherwise undertake any mathematical study in their ASL.

- 3.3 Following a difficult period, there has been a strong resurgence in the numbers taking GCE Mathematics. It is commonly felt that the present specifications are more accessible than their predecessors and this has made it more realistic for a wider population to take the subject, although evidence⁸ suggests that many centres restrict access to those with grades A and A* at GCSE. After the trauma of the initial Curriculum 2000 specifications, when entries for AGCE Mathematics dropped by a fifth, and with all the changes taking place elsewhere in the curriculum needing to be addressed, many voices are calling for there to be no changes to GCE Mathematics (and Further Mathematics).

The principle of 'if it ain't broke, don't fix it' has merit but we need to ensure that qualifications will be fit for purpose in the future as well as now.

In fact, although the uptake position has vastly improved following introduction of the current specifications it is less than clear that the outcomes are as good as they might be. The intention is to preserve the good features of the current A GCE mathematics, but deal with some of the shortcomings which arise from the six-module structure and the presence of choice; these include the

⁷ ASGCE Use of Mathematics has existed for some time, a pilot AGCE Use of Mathematics will begin in September 2008.

⁸ For example, in Matthews, A., and Pepper, D., *Evaluation of participation in GCE mathematics*, QCA, 2008.

difficulty in providing effective assessment when each examination is rather short, the lack of synoptic assessment, the lack of balance between pure and applied, and the possibility that the final module will not be taken seriously by students already reasonably sure of an A grade.

The shape of the 14–19 curriculum will change radically, particularly with the advent of the Diplomas. These changes are likely to increase the demand for mathematics teachers at a time when supply is not increasing, so there is a need for mathematics teachers to be deployed as efficiently as possible.

Further, GCEs in mathematical subjects have yet to absorb the principles of the revisions made in other subjects in 2008 and will be expected to do so from 2011. Cognisance also needs to be taken of the reforms to Key Stages 3 and 4 which are currently underway.

- 3.4 At present, GCE Mathematics consists of six units, four units forming a core of pure mathematics, two AS and two A2 units and two applied units which are chosen from, in most specifications, six units: AS and A2 units in each of Decision Mathematics, Mechanics and Statistics. In Mathematics uniquely, this arrangement permits an AGCE to contain four AS units. It also creates six routes to the qualification. This can allow the choice of applied content to be related to the learner's interests. Despite the efforts of the awarding bodies, these routes are widely not perceived as being equally demanding, and there is evidence⁹ that those choosing a route often select the route which they believe to be easiest.
- 3.5 ACME recommends that there should be no choice within AS GCE and A GCE Mathematics (neither by provision of option within specifications nor differences of content between specifications).
- All learners would be provided with an entitlement to a common body of knowledge and skills.
 - Often, choices of specification and unit are made by centres rather than learners.
 - Where learners do make choices these are often poorly informed and can have problematic consequences for progression.
 - Choice is often an attempt to seek out an easy option rather than because of a desire to study the content of a particular unit.
 - The difficulties of learners who have to change institution in mid-course would be reduced.
 - It would aid the efficient deployment of specialist and expert teachers and assessors.
 - By creating a single target population, it would assist and make more viable the development of high quality and innovative teaching resources.
 - Similarly, it would facilitate the development of high-quality assessments.
 - It would make it easier to build links between the pure and applied content.
 - All learners, regardless of centre, would take courses (and their component parts) which were equally favoured by end users.
 - Higher education would be provided with entrants with a far greater homogeneity of prior experience which would allow for greater efficiency in that sector.
 - All learners would study applied content at both AS and A2 standards.
 - GCE Mathematics and Further Mathematics would be distinct subjects without overlapping content and shared units.

Such a change would provide equality of opportunity and provide a common basis for progression. Building on this foundation, learners could, if appropriate, personalise their mathematical studies by adding GCE Further Mathematics units or qualifications, or Free Standing Mathematics Qualifications. It will be important that the applied content is presented in a way which makes it attractive to all learners pursuing GCE Mathematics.

- 3.6 A criticism that has been made of the above proposal is that if all three existing applied strands were to be incorporated into the new applied units then the coverage would be fragmentary and incoherent. We propose that only Mechanics and Statistics should be included. Decision Mathematics should no longer form part of GCE Mathematics although it could still form part of GCE Further Mathematics. (That GCE Further Mathematics could contain a distinctive form of applied mathematics would be particularly helpful to those centres where GCE Mathematics and GCE Further Mathematics are taught in parallel.) It would also be available as an FSMQ which could form part of GCE Use of Mathematics.

In making provision at this level, a balance has to be struck between breadth and depth of study. Although, there is undoubtedly much to be gained from an in-depth study of one applied strand, a

⁹ For example, in Matthews, A., and Pepper, D., *Evaluation of participation in GCE mathematics*, QCA, 2008.

strong enough case cannot be made for any one applied strand to be studied alone by all those taking GCE Mathematics. The experience of past specifications¹⁰ leads us to believe that the proposals made here would allow a balanced yet coherent provision.

The fear has been expressed that removing choice and introducing compulsory applied content (already the case for learners in many centres) will cause some learners not to take the subject. The reports that have reached us from centres which have already made this change do not suggest that this would be the case. For most learners, the choice is whether to take Mathematics or not and is little influenced by what applied units are available. Indeed, for some learners anxiously trying to choose between Mathematics and another subject, the need if they choose Mathematics to make a second decision, about their applied options, can make the other subject seem more attractive.

- 3.7 ACME recommends that GCE Mathematics should contain two AS units and two A2 units, one each of pure mathematics and applied mathematics at each standard. For both AS and A2 the balance should be 60% pure mathematics and 40% applied mathematics, the latter being split equally between mechanics and statistics. (This would increase the present applied content by a fifth which would reduce the risk of its fragmentation and would mean that each applied chunk would represent 10% of an AGCE, a model which has been implemented successfully in the past.) To make this possible there would need to be a slight reduction in the present pure content; we believe this could be readily achieved. (The present pure core is a little generously weighted at two-thirds and the reduction would not need to be a tenth. In making the change to the core the balance between AS and A2 should be reviewed, it being a common view among teachers that the material presently in the AS core needs more teaching time than that in the A2 core.) The applied content, as well as being valuable in its own right, would be related to and reinforce the pure content, something more likely to happen when all are studying the same applied material.
- 3.8 There is a widespread view that whilst the present specifications make strong demands in carrying out specific mathematical techniques, they do not emphasise sufficiently problem-solving, modelling skills and the construction and communication of mathematical arguments. The opportunity should be taken of the move to four units, with the longer examination papers which it permits, to include longer unstructured questions to give more emphasis to these skills. This benefit is independent of the proposed changes to provision for applied mathematics in GCE Mathematics and for GCE Further Mathematics and is sufficient grounds for the amalgamation of the existing pure units at AS and at A2, regardless of whether the other proposals are implemented.

Most other GCE subjects changed from six to four units from 2008, those keeping six units doing so to allow assessment of practical work. There is a danger that if GCE Mathematics were to remain at six units it would be perceived by learners as more difficult (reinforcing an existing reputation) which could dissuade some learners, especially those whose progression goals were ones where mathematics is seen as useful rather than essential, from pursuing the subject.

There is a danger that in making a sudden and substantial change in the style of examination questions, teaching and learning would fail to adjust sufficiently quickly, and the resulting negative experiences could deal a significant blow to the uptake of GCE Mathematics. This can be avoided by having a gradual change of emphasis from the present stress on specific techniques towards the skills listed above. (Although the end of the life of the mathematics qualifications introduced in 2011 is uncertain, it would not be unreasonable to plan on them applying to six cohorts.) This gradual change of emphasis could be achieved by agreeing and publishing at the outset three successive sets of criteria (and associated specifications), each lasting two years, which would have the same content but differing weightings of assessment objectives with consequent changes to the examinations (and associated specimen assessment materials). This would meet the regulatory need for stability during the lifetime of subject criteria and provide teachers and resource providers with a clear sense of the direction of travel, so making them better able to implement the desired change of emphasis.

- 3.9 The current requirement for a non-calculator unit does not sit comfortably with a four-unit structure. The Level 3 content, which should be the focus of any GCE assessment, can be adequately assessed without this restriction and the consequent need for some contrivance, such as having a unit divided into two sections with candidates picking up their calculators under their chairs part way through the examination while simultaneously invigilators collect the first non-calculator section from their desks.

ACME does not favour the introduction at this time into GCE Mathematics (or Further Mathematics) of forms of assessment other than timed written papers. Other forms of assessment would be unlikely to

¹⁰ For example, the Cambridge linear and SMP 16–19 specifications from the period immediately before the introduction of Curriculum 2000.

enhance public confidence in such high profile qualifications and would be unwelcome to most teachers, especially at a time when they are being asked to cope with other changes. Nevertheless, ACME would not wish to see learners' activities restricted to practising those types of question used in GCE examinations and would encourage the development of more open learning activities for use outside of public examinations.

- 3.10 ACME is well aware that the changes proposed above will make considerable demands on teachers and it will be vital to the success of these changes that proper provision, co-ordinated by the National Centre for Excellence in the Teaching of Mathematics, is made for professional development to support teachers in delivering the new specifications. Teachers will need support in ways of teaching which develop in learners problem-solving, modelling skills and the communication of mathematical arguments. Many teachers have not taught, or not taught recently, one or other of the applied strands and will need support in developing, or refreshing, their subject expertise and confidence in that area. The proposed changes provide a particular opportunity for promoting good practice in the teaching of GCE Mathematics and it is important that sufficient priority is given to providing the resources to do this.
- 3.11 The arguments which lead us to recommend the homogenisation of GCE Mathematics also lead us to recommend the withdrawal of GCE Pure Mathematics. The subject has a small uptake and it is a less suitable basis for progression than GCE Mathematics. Further, in order to be viable, it needs to be composed of units used in GCE Mathematics and Further Mathematics, something which can constrain the design of those subjects and would obstruct the improvements in them that we propose.
- 3.12 With falling numbers of entries, for many years the imminent death of GCE Further Mathematics was foretold, but with the introduction of the latest specifications and the advent of the Further Mathematics Network there has been a great and welcome growth in uptake. Nevertheless, the provision of the subject in many schools and colleges is still problematic and its design needs to recognise this. The difficulties that centres can experience in delivering GCE Further Mathematics would suggest that it is expedient to retain, for the 2011 revision, the present six-unit structure. Something approaching the present flexibility of choice of units should be retained, although consideration should be given to the creation of pure mathematics cores within the AS and A2 parts of GCE Further Mathematics.

Fears have been expressed that the proposed changes to A GCE Mathematics will have a serious knock on effect on the teaching and learning of AS and A GCE Further Mathematics, and that this will result in reduced take up of Further Mathematics and possibly of Mathematics itself. All except one member of ACME felt that the potential benefits of the changes that ACME is proposing outweigh the possible risks, but there was one committee member who disagreed strongly. This member has outlined his reasons for dissent in an annexe to this position statement.

- 3.13 The Further Mathematics Network has an essential and continuing role to play in ensuring that all learners in England, wherever they are educated, have access to GCE Further Mathematics. Indeed we would commend to those responsible for mathematics education in Northern Ireland and Wales consideration of developing its work in those countries. With the growing encouragement by higher education of young people aspiring to mathematically-demanding degree courses to take GCE Further Mathematics, at least to AS Level, it is ever more important to ensure nationwide provision of the subject¹¹.
- 3.14 The GCE Further Mathematics (Additional Award) has a small uptake, it adds to the complexity of the offering, and it is not clear that all its users are best served by taking more units at the same level. For some learners the pressure to cover additional units can be prejudicial to the quality of their learning, with over emphasis on examination preparation. For others, stretch and challenge may be better provided by other types of experience. Nevertheless, for some learners the study of more than six GCE Further Mathematics units is appropriate and it is desirable that such further study, be it of nine or twelve units in all, is recognised by an appropriate GCE Further Mathematics qualification.
- 3.15 Participation in ASGCE Use of Mathematics has grown steadily since it was first awarded in 2002. Although the numbers are still not large¹²; it is viewed very positively by those who teach it as a

¹¹ At present, some centres which have small numbers of learners studying GCE Further Mathematics but GCE Mathematics classes taking different applied units exploit this situation to reduce the amount of additional teaching required to offer GCE Further Mathematics by having Further Mathematics learners attend the applied lessons of more than one Mathematics class. The fear has been expressed that the proposed change in the relationship between GCE Mathematics and GCE Further Mathematics could lead to some of these centres no longer offering GCE Further Mathematics; the support of the Further Mathematics Network will be particularly valuable in supporting such centres in continuing to offer GCE Further Mathematics.

¹² For ASGCE Use of Mathematics, in 2002 there were 102 entries graded whilst in 2007 there were 1282 entries graded (Joint Council for Qualifications, *Inter-Awarding Body Statistics Winter 2006/7 and Summer 2007*, April 2008).

valuable means of widening participation in mathematics and developing a different and more practical skill set than GCE Mathematics. A deterrent to the uptake of the subject has been the lack of the opportunity to progress to an AGCE; as part of the Pathways developments, an AGCE (with revised ASGCE) is to be piloted.

- 3.16 GCE Use of Mathematics has the potential to widen participation in mathematics at Level 3 whilst providing a different skill set from GCE Mathematics and one which would be attractive to many learners and end users. Nevertheless, the introduction of GCE Use of Mathematics alongside GCE Mathematics raises several issues:
- comparability of the qualifications and standards (and how they are perceived),
 - whether centres (and which centres) will offer both qualifications (and the consequences for learners if their centre offers only one of these qualifications, which could lead to social inclusion issues),
 - rigor of and public confidence in any portfolio-based assessment in a high-stakes GCE qualification,
 - access to progression (especially to higher education),
 - effects on the prior experience and readiness of the intake into higher education (with consideration of particular types of course and institution) and
 - the relationship of Use of Mathematics with the Diplomas (and any mathematical content therein).
- 3.17 Steps will need to be taken to promote GCE Use of Mathematics as a separate qualification in its own right and to counteract any perception of it as a softer or second class mathematics GCE. (The parallel use of the title Use of Statistics for the GCE qualification in statistics may be helpful in this regard.) It is vital that learners receive appropriate information, advice and guidance in order to select appropriately between GCE Mathematics and Use of Mathematics. This could usefully form part of improved information, advice and guidance about all mathematics qualifications and their long-term value to learners. In particular, there will need to be clear statements about the suitability of both qualifications with regard to a wide range of progression routes.

Given the behaviour of learners and centres where pairs of subjects or options within subjects already exist, it will be important to monitor the provision and uptake of both GCEs, and the subsequent progression of learners. In promoting the subject to centres (individually or in consortia) and to learners, it would be prudent to seek a gradual uptake of GCE Use of Mathematics so that opportunity would arise to mitigate any undesirable effects that may arise before they have a large scale impact on mathematics education.

Up to now, ASGCE Use of Mathematics has been distinctive in its use of portfolio assessment alongside written examinations for each unit. The proposals being piloted restrict portfolio assessment to A2 and provide only one form of assessment for each unit. Whilst these changes simplify the assessment, this may be at the expense of reducing the educational value of the qualification with learners having less opportunity to develop those skills, of modelling and in the use of ICT, which are particular to GCE Use of Mathematics. Whilst standardising on one assessment component per unit may be desirable in simplifying and reducing the burden of assessment, we would counsel against a Procrustean approach which maimed qualifications in order to achieve that standardisation.

- 3.18 Many have suggested to us that in the longer term a new GCE in Mathematics might be forged out of the best in the present GCE Mathematics and GCE Use of Mathematics. ACME recommends that exploratory work be undertaken into the possibility of a future realignment of GCE qualifications in mathematics (whatever the outcomes of the 2013 review of qualifications, there will be a need for some form of accredited assessment of mathematics at this level), with a revised GCE Mathematics (formed largely out of GCE Mathematics and GCE Use of Mathematics) aimed at a wide range of users of mathematics and a revised GCE Further Mathematics aimed as an additional qualification for those with a particular interest in mathematics or where the use they wish to make of it requires a deeper knowledge of the subject. The exploratory work should include the consideration of a wider range of assessment styles than just timed written papers. Before introducing any such realignment of qualifications, assurance would be needed that the new provision would both be at least as accessible as the previous provision and provide a stimulating mathematical education for the more able.

4 Stretch, Challenge and Inspiration in Mathematics

- 4.1 There is a very wide spread of ability in mathematics, wider than most other subjects, even among those taking Level 3 qualifications. It is almost inevitable, if those qualifications are to be accessible to

most Level 3 learners that there will be a substantial number of learners for whom they will not provide sufficient stretch, challenge or inspiration.

- 4.2 From first award in 2010, GCEs in mathematical subjects alongside other subjects will have an A* grade. In mathematics, this is likely to reward a high degree of accuracy on straightforward questions rather than to reveal a high degree of aptitude for the subject. Given that, at least in the forms currently in use, mathematics questions have a particularly limited range of ability for which they are appropriate (proving inaccessibly hard or unchallenging for others); there will be inevitable limitations on what can be achieved through the introduction of more searching questions in the specifications being introduced in 2011. Trying to provide for the most able must not lead to less accessibility for others.
- 4.3 There is a growing dissatisfaction in universities with the effectiveness of GCE Mathematics as a discriminator for admissions for the more competitive courses. This has been accompanied by a growth of interest in universities setting admissions tests, singly or in combination. The growth of such tests is undesirable because of both the burden it places on learners and the resources it diverts in the education system. Questions also arise as to the social inclusiveness of admissions based on such tests, especially given the differing capacities of different centres to support learners preparing for them.
- 4.4 There needs to be an additional qualification which provides stretch, challenge and inspiration for the most mathematically able learners, and acts as an effective measure of aptitude to pursue mathematically highly demanding degree courses. Such a nationally available (and funded) qualification should make it feasible to support learners wherever they were studying through the Further Mathematics Network and the use of web-based resources.

The Advanced Extension Award (AEA) was introduced in Curriculum 2000 to fulfil this role. The uptake has been disappointing but is growing. (A particular issue has been the lack of funding and the initial lack of UCAS points.) The qualification fulfils a valuable role and it should not be withdrawn until a suitable replacement qualification is available.

The experience gained through the AEA suggests that there would be value in developing an improved replacement which would be aimed at all A grade GCE Mathematics candidates and had a grading scale which recognised achievement at a broader range of levels than the AEA (which has only two pass grades) and allowed more nuanced use by higher education selectors. It would be synoptic of the core (if recommendations elsewhere in this report were adopted this would be extended to include applied mathematics which the present AEA does not assess) and contain unstructured questions which tested problem-solving and the ability to develop and communicate mathematical arguments in greater depth than does A GCE Mathematics.

The AEA was designed not to require additional teaching, although few are successful without it. Somehow, the opportunity was thereby missed to provide a stimulus for greater stretch, challenge and inspiration of the more mathematically able during their GCE courses. Alongside any new qualification there should be initiatives to encourage the development of materials, text and web-based, which would support such stimulating experiences throughout the two years of the GCE course.

- 4.5 Although acceleration may be appropriate for a few learners, for most of the most able learners in mathematics it is far more beneficial for them to deepen and extend their mathematics at Level 3 rather than proceed prematurely to Level 4. Whilst the discussion so far in this paper has been largely of accredited qualifications, it should be remembered there are many other valuable activities, such as those of the Millennium Mathematics Project and the United Kingdom Mathematics Trust which can provide stretch, challenge and inspiration for the most able mathematical learners. Centres should include such activities within their provision as well as preparation for qualifications; these activities can be motivating and enriching for many learners, not only the most able.

Paragraph of Dissent

It is with regret that I find myself recording my dissent from recommendations (5) and (6) of the position statement of the ACME Level 3 position paper. The decision to make these recommendations pre-dates my joining ACME in September 2008.

In general, I believe that the current syllabus for mathematics at A-Level is working well, and should not be altered. It is my view that the proposals in the position statement, taken as a whole, represent a substantial change to the current curriculum and its underlying philosophy and that to implement the changes without further piloting represents a high risk.

The move from a 6 to 4 unit structure for A-Level Mathematics (while retaining a 6 module structure for A-Level Further Mathematics) would mean Mathematics and Further Mathematics cannot share any units. I believe that this complete separation between Mathematics and Further Mathematics runs the risk of reducing the numbers opting for Further Mathematics, and undoing the recent upward trends we have witnessed.

In addition I cannot support a move away from choice at AS and A-Level, forcing all students to do the same mixture of mechanics and statistics, while disallowing decision mathematics. I fear that this would result in students being less well prepared for university courses, and would narrow participation in mathematics. Moreover, I believe that the proposals for applied mathematics would result in fragmentation and loss of quality, with students seeing applied mathematics as no more than isolated topics.

Roger Porkess
ACME Member

Historical and Statistical Appendix

The General Certificate of Education was introduced in 1951. The Advanced Level was intended for approximately the most able tenth of the cohort but with the growth of post-16 education it is now taken by learners drawn from across the most able half of the cohort. From the beginning, it was possible to take one or two (or more) A Levels in mathematics, with one or more syllabuses on offer from each of the awarding bodies (of which there have been as many as ten but are now five), as well as syllabuses offered on behalf of curriculum development projects such as *Mathematics in Education and Industry* and the *School Mathematics Project*. In 1987, Advanced Supplementary Level qualifications, half the size of A Levels but of the same standard, were introduced (but were only taken by small numbers of learners). These and A Levels were replaced by Advanced Subsidiary and Advanced GCEs from 2001 as part of the Curriculum 2000 reforms; the new AS GCE being intended to be suitable for learners at the end of the first year of the A GCE course. Some modular syllabuses had begun to be introduced from the 1980s, but under Curriculum 2000 all specifications are modular, most originally with six units but from 2008 having four units.

The first core for A Level Mathematics was introduced from 1983; it contained only pure mathematics and was intended to form 40% of the syllabus. It led to overly large syllabuses which led to a decline in the numbers of learners taking mathematics; a smaller revised core was introduced in 1995. With the introduction of Curriculum 2000 (in which the norm is that in the first year learners take four GCE subjects rather than three), the core was again revised, this proved too demanding and was followed by a reduction of one-fifth in the numbers taking A GCE Mathematics. In response to this drastic fall, a revised core was introduced in 2004 which spread the existing pure content over four units instead of three and reduced the number of applied units from three to two. Since 2004, there has been a substantial and continuing growth in numbers taking A GCE Mathematics (and proportionately an even greater growth in the numbers taking A GCE Further Mathematics).

A Level GCE Entries 1989 – 2001 and A GCE Entries 2002 – 2008

| year | all subjects | all mathematical subjects | % | all mathematical subjects excluding Further Mathematics | Further Mathematics | % |
|------|--------------|---------------------------|------|---|---------------------|------|
| 1989 | 661591 | 84744 | 12.8 | separate data for Further Mathematics were not published by the JCQ before 2003 | | |
| 1990 | 684117 | 79747 | 11.7 | | | |
| 1991 | 699041 | 74972 | 10.7 | | | |
| 1992 | 731024 | 72384 | 9.9 | | | |
| 1993 | 734081 | 66340 | 9.0 | | | |
| 1994 | 732974 | 64919 | 8.9 | | | |
| 1995 | 725992 | 62188 | 8.6 | | | |
| 1996 | 739163 | 67442 | 9.1 | | | |
| 1997 | 777710 | 68880 | 8.9 | | | |
| 1998 | 794692 | 70554 | 8.9 | | | |
| 1999 | 783692 | 69945 | 8.9 | | | |
| 2000 | 771809 | 67036 | 8.7 | | | |
| 2001 | 748866 | 66247 | 8.8 | | | |
| 2002 | 701380 | 53940 | 7.7 | | | |
| 2003 | 750537 | 55917 | 7.5 | 50602 | 5315 | 10.5 |
| 2004 | 766247 | 58508 | 7.6 | 52788 | 5720 | 10.8 |
| 2005 | 783878 | 58830 | 7.5 | 52897 | 5933 | 11.2 |
| 2006 | 805698 | 63252 | 7.9 | 55982 | 7270 | 13.0 |
| 2007 | 805657 | 67965 | 8.4 | 60093 | 7872 | 13.1 |
| 2008 | 827737 | 73684 | 8.9 | 64593 | 9091 | 14.1 |

Source: Joint Council for Qualifications (previously Joint Council for General Qualifications) communicated by Roger Porkess. These figures include all entries in the United Kingdom. The thicker horizontal bars indicate the introduction of new specifications following from revisions to the core.

The Government target of 56000 for participation in A GCE Mathematics for 16 to 19 year olds in England by 2014 was reached this year and has been raised to 70000.